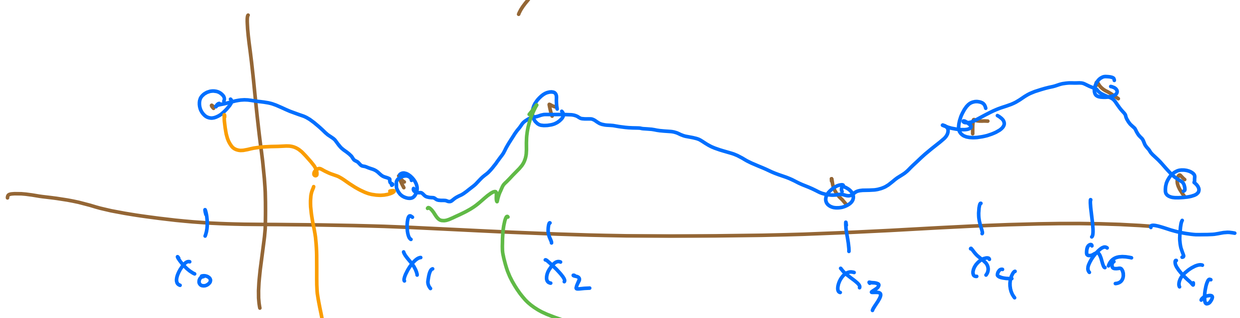
New kind of interpolation between data points.

→ Natural Cubic Spline.

Given $[x_0, y_0], [x_1, y_1], [x_2, y_2], \dots, [x_N, y_N]$



$f_0(x) = A_0 x^3 + B_0 x^2 + C_0 x + D$

$f_1(x) = A_1 x^3 + B_1 x^2 + C_1 x + D_1$

. . . .

... $f_j(x) = A_j x^3 + B_j x^2 + C_j x + D_j$

Constraints to make the cubic spline smooth.

① fcn $f_j(x)$ must satisfy

for $0 \le j \le N-1$ $\quad f_j(x_j) = y_j$, $\quad f_j(x_{j+1}) = y_{j+1}$

(so that the jth fcn hits (x_j, y_j) & (x_{j+1}, y_{j+1}).)

② for $1 \le j \le N-1$

$$f'_{j-1}(x_j) = f'_j(x_j)$$

$$f''_{j-1}(x_j) = f''_j(x_j)$$

(3) End points

$$f''_0(x_0) = 0$$

$$f''_{N-1}(x_N) = 0$$

We have $4N$ unknowns A_j, B_j, C_j, D_j

$0 \leq j \leq N-1$

From (1) $2N$ equations
(2) $2N-2$ equations
(3) 2 equations

$\Bigg\}$ $4N$ equations.

$4N$ equations & $4N$ unknowns.

↳ one can prove you get a single unique solution.

Example on sagemath.

Note: The cubic spline through the data points is the unique curve $y = f(x)$ that has the minimum value of

$$\int_{\min x}^{\max x} (f''(x))^2 \, dx$$

"least curved" function.